Radiation-Reaction Forces in the Hydrogen Atom, and in the Neutron

H. Vic Dannon
vic0@comcast.net
September, 2015

Abstract The loss of Radiation Energy by an accelerated charge manifests in the Abraham-Lorentz Radiation-Dumping equation as a Radiation-Reaction frictional force.

A Hydrogen electron orbiting with angular velocity ω is accelerated towards the center at

$$\vec{a} = \ddot{\vec{x}} = \frac{d^2\vec{x}}{dt^2}$$

by an electro-magnetic spring-like force

$$k\vec{x} = m\omega^2\vec{r}_{\mathrm{H}}.$$

Then, the Abraham-Lorentz equation is

$$m\vec{a} - \vec{F}_{\text{radiation reaction}} + m\omega^2\vec{r}_{\mathrm{H}} = 0.$$

And we show that the Radiation-Reaction force is

$$\vec{F}_{\text{radiation reaction}} = \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{\vec{a}}$$

A correct derivation for this $\vec{F}_{\text{radiation reaction}}$ was not given before, and we supply the derivation here.

We compute the weight of $\vec{F}_{\text{radiation reaction}}$ in the Abraham-Lorentz equations for the Hydrogen's electron, the Hydrogen's proton, the Neutron's electron, and the Neutron's proton.

While that force is negligible, we show that the lost radiation energy is crucial to the existence of the electron-proton system: without radiation from the proton towards the electron, the electron spirals onto the proton in split seconds, and without radiation from the electron towards the proton, the proton spirals onto the electron in split seconds.

We show that for both the Hydrogen, and the Neutron,

the Radiation Reaction Force on the Proton is

$$\sim 6.5 \times \text{(Radiation Reaction Force on the Electron)}$$

We resolve the paradox of "the Runaway Solution" to the Abraham-Lorentz equation. We show that such solution violates Newton's Second Law, and does not exist.

Radiation damping widens the spectral line generated by the transition of an electron from a higher orbit to a lower one: We recall that at angular frequency $\omega = \omega_0$, the broadened spectral line bell has Mean Duration

$$\Delta t = \frac{1}{\Gamma},$$

And at half the spectral line height, its width is

$$\Delta\omega = \Gamma .$$

where $\Gamma\dot{x}$ is the radiation reaction term in $\ddot{x} + \Gamma\dot{x} + \omega_0^2 x = 0$.

Keywords: Electromagnetic Radiation of Accelerated Charge, Relativistic, Radiation Loss, Quantized Angular Momentum, Atomic Orbits, Electron, Proton, Atom, Nucleus Radius,

Physics & Astronomy Classification Scheme: 41.60-m; 32; 32.10-f; 31.15.B-;

Contents

The Abraham-Lorentz Radiation-Damping Equation vs. Bohr's Loss-less Equation

At the beginning of the 20th century, Abraham and Lorentz had different models for a moving electron. Abraham assumed that during its motion the electron remained a rigid sphere, while Lorentz allowed that sphere to contract into an ellipsoid in the direction of the motion.

In either model, if the electron is accelerated at

$$\vec{a} = \ddot{\vec{x}} = \frac{d^2\vec{x}}{dt^2}$$

by an electro-magnetic spring-like force

$$k\vec{x},$$

against the frictional Radiation-Reaction force

$$\vec{F}_{\text{radiation reaction}}$$

then, the Abraham-Lorentz equation of motion for the electron is

$$m\vec{a} - \vec{F}_{\text{radiation reaction}} + k\vec{x} = 0.$$

The radiation power loss by the electron

$$\frac{dP}{dt} = -\frac{e^2}{6\pi\varepsilon_0 c^3}\vec{a}^2,$$

equals

$$\frac{\vec{F}_{\text{radiation reaction}} \cdot d\vec{x}}{dt} = \vec{F}_{\text{radiation reaction}} \cdot \vec{\upsilon}.$$

Thus,

$$\vec{F}_{\text{radiation reaction}} \cdot \vec{\upsilon} = -\frac{e^2}{6\pi\varepsilon_0 c^3}\vec{a}^2.$$

Previous attempts to derive an explicit formula for

$$\vec{F}_{\text{radiation reaction}}$$

have failed, exposing ignorance of Arithmetic. We shall supply a correct derivation for an electron-proton system in the next section.

For the Hydrogen's electron, the accelerating force is the electric attraction to the proton

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_{\text{H}}^2}\vec{1}_{\vec{r}_{\text{H}}}$$

The equality of the accelerating force to the centripetal force

$$m\vec{a} - m\omega^2\vec{r}_{\text{H}} = 0$$

is Bohr's Loss-less equation.

Then, the electric force balances

$$m\omega^2\vec{r}_{\text{H}}.$$

The equation that includes the Radiation Reaction force

$$m\vec{a} - \vec{F}_{\text{radiation reaction}} - m\omega^2\vec{r}_{\text{H}} = 0,$$

is the Abraham Lorentz Frictional Equation.

Then, the electric force balances

$$\vec{F}_{\text{radiation reaction}} + m\omega^2\vec{r}_{\text{H}}$$

1.

The Radiation-Reaction Force on a Charge in a Circular Motion

1.1 A Bizarre Claim that if a Sum is Zero, Each of its Terms is Zero

Integrating by parts the equation

$$\vec{F}_{\text{radiation reaction}} \cdot \vec{\upsilon} = -\frac{e^2}{6\pi\varepsilon_0 c^3}\vec{a}^2 ,$$

over a period of electron's motion T, textbooks obtain

$$\int_{t=0}^{t=T}\left(\vec{F}_{\text{radiation reaction}} - \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{\vec{a}}\right)\cdot\vec{\upsilon}dt = 0 .$$

Namely, the summation

$$\sum_{t=0}^{t=T}\left(\vec{F}_{\text{radiation reaction}} - \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{\vec{a}}\right)\cdot\vec{\upsilon}dt = 0$$

From this they bizarrely conclude that

$$\vec{F}_{\text{radiation reaction}} - \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{\vec{a}} = 0$$

at any given time during the period.

To see how bizarre is that conclusion, subtract

$$\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots=1,$$

from

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots.=1.$$

Then,

$$\left(\frac{1}{2}-(1-\frac{1}{2})\right)+\left(\frac{1}{4}-(\frac{1}{2}-\frac{1}{3})\right)+\left(\frac{1}{8}-(\frac{1}{3}-\frac{1}{4})\right)+\ldots=0.$$

Can we conclude that

$$\left(\frac{1}{4}-(\frac{1}{2}-\frac{1}{3})\right)=0?$$

$$\left(\frac{1}{8}-(\frac{1}{3}-\frac{1}{4})\right)=0?$$

$$\left(\frac{1}{16}-(\frac{1}{4}-\frac{1}{5})\right)=0?$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

We supply here the missing proof to the

1.2 Abraham-Lorenz Formula

In the Circular Motion of a Charge e

$$\boxed{\vec{F}_{\text{radiation reaction}}=\frac{e^2}{6\pi\varepsilon_0 c^3}\dot{\vec{a}}}$$

Proof:

$$\vec{F}_{\text{radiation reaction}} \cdot \vec{\upsilon} = -\frac{e^2}{6\pi\varepsilon_0 c^3}\vec{a}\cdot\vec{a}$$

$$= -\frac{e^2}{6\pi\varepsilon_0 c^3}\dot{\vec{\upsilon}}\cdot\vec{a}$$

$$= -\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{d}{dt}(\vec{\upsilon}\cdot\vec{a}) - \vec{\upsilon}\cdot\dot{\vec{a}}\right)$$

In a circular motion,

$$\vec{\upsilon} \perp \vec{a},$$

$$\vec{\upsilon}\cdot\vec{a} = 0.$$

Therefore,

$$\vec{F}_{\text{radiation reaction}} \cdot \vec{\upsilon} = \frac{e^2}{6\pi\varepsilon_0 c^3}\vec{\upsilon}\cdot\dot{\vec{a}},$$

$$\left(\vec{F}_{\text{radiation reaction}} - \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{\vec{a}}\right)\cdot\vec{\upsilon} = 0.$$

Since at any time t, $\vec{\upsilon}(t) \neq 0$,

$$\vec{F}_{\text{radiation reaction}} = \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{\vec{a}}.\square$$

2.

The Damped Harmonic Motion of a Charge in a Circular Motion

The Abraham-Lorentz Equation for a charge in a circular motion is

$$m\ddot{x} - \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{a} + m\omega^2 x = 0$$

We want to show that the spring term

$$m\omega^2 x\,,$$

dominates the damping term

$$\frac{e^2}{6\pi\varepsilon_0 c^3}\dot{a}\,,$$

so that the solution of the damped equation is close to the solution of the loss-less equation

$$m\ddot{x} + m\omega^2 x = 0\,.$$

To that end we need to approximate

$$\dot{a} = \dddot{x}$$

in terms of $x(t)$, which we don't know.

To obtain that approximation for $\dot{a} = \dddot{x}$, we iterate the solution of the damped equation by the solution of the loss-less equation

$$x(t) = x_0 e^{i(\omega t + \phi)}.$$

Then,

$$\dot{x}(t) = i\omega x_0 e^{i(\omega t + \phi)},$$

$$\ddot{x}(t) = -\omega^2 x_0 e^{i(\omega t + \phi)},$$

$$\dddot{x}(t) = -i\omega^3 x_0 e^{i(\omega t + \phi)}.$$

Thus, we approximate $\dfrac{F_{\text{radiation reaction}}}{m\omega^2 x}$ by

$$\frac{\dfrac{e^2}{6\pi\varepsilon_0 c^3}\dot{a}}{m\omega^2 x} \sim \frac{\dfrac{e^2}{6\pi\varepsilon_0 c^3}\omega^3 x_0 e^{i(\omega t + \phi)}}{m\omega^2 x}$$

$$= \frac{e^2\omega}{6\pi\varepsilon_0 c^3 m}$$

2.1
$$\frac{e^2\omega}{6\pi\varepsilon_0 c^3 m}$$

approximates the weight of the Radiation Damping Force in the Abraham Lorentz Equation

3.

The Radiation-Reaction Force on the Hydrogen's Electron

3.1 The Abraham-Lorentz Equation for the Hydrogen's Electron

$$m\ddot{x} - \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{a} + m\omega_{\mathrm{H}}^2 x = 0,$$

where from [Dan1, p.47], the Hydrogen's electron angular velocity is

$$\omega_{\mathrm{H}} = 4.1334366 \times 10^{16} \text{radians / sec}$$

3.2 *The Weight of Radiation Damping in the Hydrogen Electron equation is*

$$(2.584815483) \times 10^{-7}$$

<u>*Proof*</u>:

$$\frac{e^2\omega_{\mathrm{H}}}{6\pi\varepsilon_0 c^3 m} = \frac{(1.60217733)^2(10^{-19})^2(4.1334366)(10^{16})}{6\pi(8.854187817)10^{-12}(9.1093897)10^{-31}(3)^3(10^8)^3}$$

$$= (2.584815483) \times 10^{-7}. \square$$

While the Radiation-Reaction Force in the equation seems negligible, the radiated energy is crucial to the existence of the proton-electron system. Without the energy radiated by the proton onto the electron, the electron would spiral onto the proton in a split second:

3.3 *If not for compensating radiation from the Hydrogen's Proton, the Electron would spiral onto the Proton in*

$$\approx 1.57 \times 10^{-11} \text{ sec.}$$

<u>*Proof*</u>: Since

$$m\frac{v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2},$$

The kinetic energy of the electron in its circular orbit is

$$\tfrac{1}{2}mv^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}.$$

The electric energy of the Hydrogen Atom is

$$-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}.$$

Therefore, the total electron energy is

$$-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r},$$

and its rate of change is

$$\frac{d}{dt}\left\{-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right\} = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\frac{dr}{dt}.$$

This equals the rate at which the electron radiates energy as it accelerates towards the proton,

$$\left[\frac{e^2}{6\pi\varepsilon_0 c^3}a^2\right]_{t-\frac{r_{\mathrm{H}}}{c}},$$

where

$$r_{\mathrm{H}} = 5.29277249\times 10^{-11}\mathrm{m} \sim 5.3\times 10^{-11}\mathrm{m}.$$

The retarded time is

$$t-\frac{r_{\mathrm{H}}}{c} \approx t-\frac{5.3\times 10^{-11}}{3\times 10^{8}} \sim t-\frac{1.77}{10^{19}} \approx t,$$

and the acceleration of the electron is

$$a = \frac{[v\gamma(v)]^2}{r} \approx \frac{v^2}{r}.$$

Therefore, the Electron's Radiation rate is

$$\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{v^2}{r}\right)^2 \approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)^2.$$

Since it equals the rate of change of the electron's energy,

$$\frac{\cancel{e}^2}{6\cancel{\pi}\cancel{\varepsilon}_0 c^3}\left(\frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)^2 \approx \frac{1}{2}\frac{1}{4\cancel{\pi}\cancel{\varepsilon}_0}\frac{\cancel{e}^2}{r^2}\frac{dr}{dt},$$

$$dt \approx \frac{3}{4}c^3\left(4\pi\varepsilon_0 \frac{m}{e^2}\right)^2 r^2 dr\,,$$

where

$$m = 9.1093897 \times 10^{-31}\text{Kg} \sim 9.11 \times 10^{-31}\text{Kg}\,,$$

$$e = 1.60217733 \times 10^{-19}\text{C} \sim 1.6 \times 10^{-19}\text{C}$$

Therefore, the electron will spiral into the proton in

$$t\Big|_{r=0}^{r=5.3\times10^{-11}} \approx \frac{1}{4}c^3\left(4\pi\varepsilon_0 \frac{m}{e^2}\right)^2 r^3\Big|_{r=0}^{r=5.3\times10^{-11}}\textbf{sec}$$

$$\approx \underbrace{\frac{1}{4}c^3(\underbrace{\frac{4\pi}{\mu_0}}_{10^7}\underbrace{\mu_0\varepsilon_0}_{c^{-2}})^2}_{\frac{1}{4c}10^{14}}\left(\frac{m}{e^2}\right)^2(5.3)^3 10^{-33}$$

$$\approx \frac{1}{12\cdot 10^8}10^{14}\frac{(9.11\cdot10^{-31})^2}{(1.6\cdot10^{-19})^4}(5.3)^3\cdot10^{-33}$$

$$\approx \frac{1}{12}\cdot\frac{(9.11)^2}{(1.6)^4}(5.3)^3 10^{-13}$$

$$\approx 1.57\times10^{-11}\textbf{sec.}\square$$

4.

The Radiation-Reaction Force on the Hydrogen's Proton

4.1 The Abraham-Lorentz Equation for the Hydrogen's Proton

$$M\ddot{X} - \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{A} + M\Omega_{\mathrm{H}}^2 X = 0,$$

where from [Dan1, p.38], the Hydrogen's Proton angular velocity is

$$\Omega_{\mathrm{H}} = 2.705755062 \times 10^{17} \text{ radians/sec}$$

4.2 *The Weight of Radiation Damping in the Hydrogen's Proton equation is*

$$(8.159182866) \times 10^{-9}$$

<u>*Proof*</u>:

$$\frac{e^2\Omega_{\mathrm{H}}}{6\pi\varepsilon_0 c^3 M_{\mathrm{p}}} = \frac{(1.60217733)^2(10^{-19})^2(2.705755062)10^{17}}{6\pi(8.854187817)10^{-12}(3)^3(10^8)^3(1.6726231)10^{-27}}$$

$$= (8.159182866) \times 10^{-9}. \square$$

While the Radiation-Reaction Force in the equation seems negligible, the radiated energy is crucial to the existence of the proton-electron system. Without the energy radiated by the electron onto the proton, the proton would spiral onto the electron in a split second:

4.3 *If not for compensating radiation from the Hydrogen's Electron, the Proton would spiral onto the Electron in*

$$\approx 1.8 \times 10^{-8}\,\text{sec.}$$

<u>*Proof*</u>: Since
$$M\frac{V^2}{\rho} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2},$$

The kinetic energy of the proton in its circular orbit is

$$\tfrac{1}{2}MV^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho}.$$

The electric energy of the Electron-Proton Atom is

$$-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho}.$$

Therefore, the total proton energy is

$$-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho},$$

and its rate of change is

$$\frac{d}{d\tau}\left\{-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho}\right\} = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\frac{d\rho}{d\tau}.$$

This equals the rate at which the proton radiates energy as it accelerates towards the electron,

$$\left[\frac{e^2}{6\pi\varepsilon_0 c^3}A^2\right]_{\tau-\frac{\rho_{\mathrm{H}}}{c}}.$$

By [Dan2, p.21],

$$\rho_{\mathrm{H}} = 1.221173735\times 10^{-12} \sim 1.2\times 10^{-12}$$

Then, the retarded time is

$$\tau-\frac{\rho_{\mathrm{H}}}{c} \approx \tau-\frac{1.2\times 10^{-12}}{3\times 10^{8}} \sim \tau-\frac{4}{10^{19}} \approx \tau\,.$$

The acceleration of the proton is

$$A = \frac{[V\gamma(V)]^2}{\rho} \approx \frac{V^2}{\rho}.$$

Thus, the Proton's Radiation rate is

$$\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{V^2}{\rho}\right)^2 \approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{M}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\right)^2.$$

Since it equals the rate of change of the proton's energy,

$$\frac{\cancel{e}^2}{6\cancel{\pi}\cancel{\varepsilon}_0 c^3}\left(\frac{1}{M}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\right)^2 \approx \frac{1}{2}\frac{1}{4\cancel{\pi}\cancel{\varepsilon}_0}\frac{\cancel{e}^2}{\rho^2}\frac{d\rho}{d\tau},$$

$$d\tau \approx \frac{3}{4}c^3\left(4\pi\varepsilon_0\frac{M}{e^2}\right)^2\rho^2 d\rho\,,$$

Where

$$M = 1.6726231\times 10^{-27}\,\text{Kg} \sim 1.7\times 10^{-27}\,\text{Kg}\,,$$

$$e = 1.60217733\times 10^{-19}\,\text{C} \sim 1.6\times 10^{-19}\,\text{C}$$

Therefore, the proton will spiral into the electron in

$$\tau\Big|_{\rho=0}^{\rho=1.2\times 10^{-12}} \approx \frac{1}{4}c^3\left(4\pi\varepsilon_0\frac{M}{e^2}\right)^2\rho^3\Big|_{\rho=0}^{\rho=1.2\times 10^{-12}}\ \text{sec}$$

$$\approx \underbrace{\frac{1}{4}c^3(\frac{4\pi}{\underbrace{\mu_0}_{10^7}}\underbrace{\mu_0\varepsilon_0}_{c^{-2}})^2}_{\frac{1}{4c}10^{14}}\left(\frac{M}{e^2}\right)^2(1.2)^3 10^{-36}$$

$$\approx \frac{1}{12\cdot 10^8}10^{14}\frac{(1.7\cdot 10^{-27})^2}{(1.6\cdot 10^{-19})^4}(1.2)^3\cdot 10^{-36}$$

$$\approx \frac{1}{12}\cdot\frac{(9.11)^2}{(1.6)^4}(1.2)^3 10^{-8}$$

$$\approx 1.8\times 10^{-8}\,\text{sec.}\square$$

5.

The Radiation-Reaction Force on the Neutron's Electron

5.1 The Abraham-Lorentz Equation for the Neutron's Electron

$$m\ddot{x} - \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{a} + m\omega_{\mathrm{e}}^2 x = 0\,,$$

where from [Dan1, p.46], the Neutron's Electron angular velocity is

$$\omega_{\mathrm{e}} = 5.485642074 \times 10^{20} \text{ radians/sec}$$

5.2 *The Weight of Radiation Damping in the Neutron's Electron equation is*

$$(3.430407659) \times 10^{-3}$$

<u>*Proof*</u>:

$$\frac{e^2\omega_{\mathrm{e}}}{6\pi\varepsilon_0 c^3 m} = \frac{(1.60217733)^2(10^{-19})^2(5.485642074)10^{20}}{6\pi(8.854187817)10^{-12}(3)^3(10^8)^3(9.1093897)10^{-31}}$$

$$= (3.430407659) \times 10^{-3}\,.\square$$

While the Radiation-Reaction Force in the electron equation seems negligible, the radiated energy is crucial to the existence of the proton-electron. Without the energy radiated by the neutron's proton onto the neutron's electron, the electron would spiral onto the proton in a split second:

5.3 *If not for compensating radiation from the Neutron's Proton, the Neutron's Electron would spiral onto the Neutron's Proton in*

$$\approx 8.77 \times 10^{-20}\,\text{sec.}$$

<u>*Proof*</u>: Since $$\frac{m}{\sqrt{1-\beta_e^2}}\frac{v_e^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2},$$

The kinetic energy of the electron in its circular orbit is

$$\tfrac{1}{2}\frac{m}{\sqrt{1-\beta_e^2}}v_e^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}.$$

The electric energy of the Electron-Proton Neutron is

$$-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}.$$

Therefore, the total electron energy is

$$-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r},$$

and its rate of change is

$$\frac{d}{dt}\left\{-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right\} = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\frac{dr}{dt}.$$

This equals the rate at which the electron radiates energy as it accelerates towards the proton,

$$\left[\frac{e^2}{6\pi\varepsilon_0 c^3}a^2\right]_{t-\frac{r_N}{c}}.$$

By [Dan1, p. 45],

$$r_N \sim 9.398741807\times 10^{-14}\text{m} \sim 9.4\times 10^{-14}\text{m}$$

Thus, the retarded time is

$$t-\frac{r_N}{c} \approx t-\frac{9.398741807\times 10^{-14}}{3\times 10^8} \sim t-\frac{3.133}{10^{22}} \approx t,$$

The acceleration of the electron is

$$a = \frac{[v_e\gamma(v)]^2}{r} \approx \frac{v_e^2}{r}.$$

Thus, the Electron's Radiation rate is

$$\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{v_e^2}{r}\right)^2 \approx \frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)^2.$$

Since it equals the rate of change of the electron's energy,

$$\frac{\cancel{e}^2}{6\cancel{\pi}\cancel{\varepsilon}_0 c^3}\left(\frac{1}{m}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)^2 \approx \frac{1}{2}\frac{1}{4\cancel{\pi}\cancel{\varepsilon}_0}\frac{\cancel{e}^2}{r^2}\frac{dr}{dt},$$

$$dt \approx \frac{3}{4}c^3\left(4\pi\varepsilon_0\frac{m}{e^2}\right)^2 r^2 dr \ ,$$

where

$$m = 9.1093897\times10^{-31}\text{Kg} \sim 9.11\times10^{-31}\text{Kg}\,,$$

$$e = 1.60217733\times10^{-19}\text{C} \sim 1.6\times10^{-19}\text{C}$$

Therefore, without compensating radiation from the neuron's Proton, the neutron's electron will spiral into the proton in

$$t\Big|_{r=0}^{r=9.4\times10^{-14}} \approx \frac{1}{4}c^3\left(4\pi\varepsilon_0\frac{m}{e^2}\right)^2 r^3\Big|_{r=0}^{r=9.4\times10^{-14}} \ \textbf{sec}$$

$$\approx \underbrace{\frac{1}{4}c^3(\frac{4\pi}{\underbrace{\mu_0}_{10^7}}\underbrace{\mu_0\varepsilon_0}_{c^{-2}})^2}_{\frac{1}{4c}10^{14}}\left(\frac{m}{e^2}\right)^2(9.4)^3 10^{-42}$$

$$\approx \frac{1}{12\cdot10^8}10^{14}\frac{(9.11\cdot10^{-31})^2}{(1.6\cdot10^{-19})^4}(9.4)^3\cdot10^{-42}$$

$$\approx \frac{1}{12}\cdot\frac{(9.11)^2}{(1.6)^4}(9.4)^3 10^{-22}$$

$$\approx 8.77\times10^{-20}\ \textbf{sec.}\square$$

6.

The Radiation-Reaction Force on the Neutron's Proton

6.1 The Abraham-Lorentz Equation for the Neutron's Proton

$$M\ddot{X} - \frac{e^2}{6\pi\varepsilon_0 c^3}\dot{A} + M\Omega_{\mathrm{p}}^2 X = 0,$$

where from [Dan1, p.38], the Neutron's Proton angular velocity is

$$\Omega_{\mathrm{p}} = 3.577789504 \times 10^{21} \text{ radians/sec}$$

6.2 *The Weight of Radiation Damping in the Neutron's Proton equation is* $(1.218496378) \times 10^{-5}$

<u>*Proof*</u>:

$$\frac{e^2\Omega_{\mathrm{p}}}{6\pi\varepsilon_0 c^3 M_{\mathrm{p}}} = \frac{(1.60217733)^2(10^{-19})^2(3.577789504)10^{21}}{6\pi(8.854187817)10^{-12}(3)^3(10^8)^3(1.6726231)10^{-27}}$$

$$= (1.218496378) \times 10^{-5}. \square$$

While the Radiation-Reaction Force in the proton's equation seems negligible, the radiated energy is crucial to the existence of the proton-electron neutron system. Without the energy radiated by the neutron's electron onto the neutron's proton, the neutron's proton would spiral onto the neutron's electron in a split second:

6.3 *If not for compensating radiation from the Neutron's electron, the Neutron's Proton would spiral onto the Neutron's Electron in*

$$\approx 1.1 \times 10^{-16}\,\text{sec}.$$

<u>*Proof*</u>: Since
$$M\frac{V^2}{\rho} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2},$$

The kinetic energy of the Neutron's proton in its circular orbit is

$$\tfrac{1}{2}MV^2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho}.$$

The electric energy of the Electron-Proton Neutron is

$$-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho}.$$

Therefore, the total proton energy is

$$-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho},$$

and its rate of change is

$$\frac{d}{d\tau}\left\{-\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho}\right\}=\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\frac{d\rho}{d\tau}.$$

This equals the rate at which the proton radiates energy as it accelerates towards the electron,

$$\left[\frac{e^2}{6\pi\varepsilon_0 c^3}A^2\right]_{\tau-\frac{\rho_{\mathrm{p}}}{c}}.$$

By [Dan1, p.32],

$$\rho_{\mathrm{p}}=2.209505336\times 10^{-15}\sim 2.2\times 10^{-15}$$

Then, the retarded time is

$$\tau-\frac{\rho_{\mathrm{H}}}{c}\approx\tau-\frac{2.2\times 10^{-15}}{3\times 10^{8}}\sim\tau-\frac{7.3}{10^{22}}\approx\tau.$$

The acceleration of the proton is

$$A=\frac{[V\gamma(V)]^2}{\rho}\approx\frac{V^2}{\rho}.$$

Thus, the Neutron's Proton Radiation rate is

$$\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{V^2}{\rho}\right)^2\approx\frac{e^2}{6\pi\varepsilon_0 c^3}\left(\frac{1}{M}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\right)^2.$$

Since it equals the rate of change of the proton's energy,

$$\frac{\cancel{e}^2}{6\cancel{\pi}\cancel{\varepsilon}_0 c^3}\left(\frac{1}{M}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\rho^2}\right)^2 \approx \frac{1}{2}\frac{1}{4\cancel{\pi}\cancel{\varepsilon}_0}\frac{\cancel{e}^2}{\rho^2}\frac{d\rho}{d\tau},$$

$$d\tau \approx \frac{3}{4}c^3\left(4\pi\varepsilon_0\frac{M}{e^2}\right)^2\rho^2 d\rho,$$

Where

$$M = 1.6726231\times 10^{-27}\,\text{Kg} \sim 1.7\times 10^{-27}\text{Kg},$$

$$e = 1.60217733\times 10^{-19}\,\text{C} \sim 1.6\times 10^{-19}\,\text{C}$$

Therefore, the neutron's proton will spiral into the neutron's electron in

$$\tau\Big|_{\rho=0}^{\rho=2.2\times 10^{-15}} \approx \frac{1}{4}c^3\left(4\pi\varepsilon_0\frac{M}{e^2}\right)^2\rho^3\Big|_{\rho=0}^{\rho=2.2\times 10^{-15}}\ \text{sec}$$

$$\approx \underbrace{\frac{1}{4}c^3(\underbrace{\frac{4\pi}{\mu_0}}_{10^7}\underbrace{\mu_0\varepsilon_0}_{c^{-2}})^2}_{\frac{1}{4c}10^{14}}\left(\frac{M}{e^2}\right)^2(2.2)^3 10^{-45}$$

$$\approx \frac{1}{12\cdot 10^8}10^{14}\frac{(1.7\cdot 10^{-27})^2}{(1.6\cdot 10^{-19})^4}(2.2)^3\cdot 10^{-45}$$

$$\approx \frac{1}{12}\cdot\frac{(9.11)^2}{(1.6)^4}(2.2)^3 10^{-17}$$

$$\approx 1.1\times 10^{-16}\,\text{sec.}\square$$

7.

Electron's Radiation-Reaction Force vs. Proton's-Radiation Reaction Force

7.1 *Radiation Reaction Force on the Hydrogen Proton is $\sim 6.5\ \times$(Radiation Reaction Force on the Hydrogen Electron)*

<u>*Proof*</u>: $$\left| \frac{\dfrac{e^2}{6\pi\varepsilon_0 c^3}\Omega_{\mathrm{H}}^3 \rho_{\mathrm{H}} e^{i(\omega t+\phi)}}{\dfrac{e^2}{6\pi\varepsilon_0 c^3}\omega_{\mathrm{H}}^3 r_{\mathrm{H}} e^{i(\omega t+\phi)}} \right| = \frac{\Omega_{\mathrm{H}}^3 \rho_{\mathrm{H}}}{\omega_{\mathrm{H}}^3 r_{\mathrm{H}}},$$

where

$r_{\mathrm{H}} =$ the Hydrogen electron orbit radius,

$\rho_{\mathrm{H}} =$ the Hydrogen proton orbit radius.

By [Dan2, p.22] $\dfrac{\rho_{\mathrm{H}}}{r_{\mathrm{H}}} = \sqrt{\dfrac{m}{M}}$; $\dfrac{\Omega_{\mathrm{H}}}{\omega_{\mathrm{H}}} = \left(\dfrac{M}{m}\right)^{\frac{1}{4}}$

$$= \left(\frac{M}{m}\right)^{\frac{3}{4}} \left(\frac{m}{M}\right)^{\frac{1}{2}}$$

$$= \left(\frac{M}{m}\right)^{\frac{1}{4}}$$

$$= \sqrt[4]{1836}$$

$$= 6.545881555$$

7.2 *Radiation Reaction Force on the Neutron's Proton is $\sim 6.5\ \times$(Radiation Reaction Force on the Neutron's Electron)*

<u>*Proof*</u>: $$\left| \frac{\frac{e^2}{6\pi\varepsilon_0 c^3}\Omega_{\mathrm{p}}^3 \rho_{\mathrm{p}} e^{i(\omega t+\phi)}}{\frac{e^2}{6\pi\varepsilon_0 c^3}\omega_{\mathrm{e}}^3 r_{\mathrm{N}} e^{i(\omega t+\phi)}} \right| = \frac{\Omega_{\mathrm{p}}^3 \rho_{\mathrm{p}}}{\omega_{\mathrm{e}}^3 r_{\mathrm{N}}},$$

where

$r_{\mathrm{N}} =$ the Neutron's electron orbit radius,

$\rho_{\mathrm{p}} =$ the Neutron's proton orbit radius.

By [Dan1, p.18], $$\frac{\rho_{\mathrm{p}}}{r_{\mathrm{N}}} = \left(\frac{m}{M}\right)^{\frac{1}{2}} \left(\frac{1-\beta_{\mathrm{p}}^2}{1-\beta_{\mathrm{e}}^2}\right)^{\frac{1}{4}}.$$

And by [Dan1, p.66], $$\frac{\Omega_{\mathrm{p}}^3}{\omega_{\mathrm{e}}^3} = \left(\frac{M}{m}\right)^{\frac{3}{4}} \left(\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}\right)^{\frac{3}{8}}.$$

Therefore,

$$\frac{\Omega_{\mathrm{p}}^3 \rho_{\mathrm{p}}}{\omega_{\mathrm{e}}^3 r_{\mathrm{N}}} = \left(\frac{M}{m}\right)^{\frac{1}{4}} \left(\frac{1-\beta_{\mathrm{e}}^2}{1-\beta_{\mathrm{p}}^2}\right)^{\frac{1}{8}}$$

$$\approx \left(\frac{M}{m}\right)^{\frac{1}{4}}$$

$$= \sqrt[4]{1836}$$

$$= 6.545881555$$

8.

The Runaway Solution for the Abraham-Lorentz equation that Does Not Exist

The Abraham-Lorentz Frictional Equation

$$m\vec{a} - \vec{F}_{\text{radiation reaction}} + m\omega^2\vec{r} = eEe^{i\omega t}$$

says that the electron e is accelerated to $\vec{a}$, by an electromagnetic Force opposed by the radial Force $m\omega^2\vec{r}$, and by an external radiation field $Ee^{i\omega t}$ that opposes the radiation reaction force $\vec{F}_{\text{radiation reaction}}$, and compensates for the radiation damped by the electron.

The electromagnetic force may be the electric attraction to the Hydrogen proton, $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_{\text{H}}^2}$, or it may be the Lorentz Force $e\vec{\upsilon}\times\vec{B}$ of a Magnetic Induction $\vec{B}$.

With no electromagnetic force, the circular motion stops, and the radial force vanishes.

With no external radiation field, an accelerating electron will keep loosing radiation energy.

Thus, with no electromagnetic force, and with no external radiation field, an accelerating electron will satisfy the equation

$$m\vec{a} - \vec{F}_{\text{radiation reaction}} = 0\,,$$

$$ma = m\underbrace{\frac{e^2}{m6\pi\varepsilon_0 c^3}}_{\tau}\dot{a}\,,$$

$$a = \tau\dot{a}\,,$$

$$a = e^{\frac{1}{\tau}t}\,.$$

That is, if the electron will accelerate, without an electromagnetic force, and without an external radiation field, we will obtain a runaway solution with exponential acceleration.

By Newton's Second Law, that will never happen: Without an external force, or energy of any kind, the acceleration must be zero, and the runaway solution does not exist.

9.

Radiation-Damping Widening of Spectral Lines

We recall the role of radiation reaction in the widening of a spectral line:

A bound electron radiating energy is described by

$$m\ddot{x} - \frac{e^2}{6\pi\varepsilon_0 c^3}\dddot{x} + m\omega_0^2 x = 0.$$

Assuming $\ddot{x} + \omega_0^2 x \approx 0$, implies $\dddot{x} \approx -\omega_0^2 \dot{x}$, and

$$\ddot{x} + \underbrace{\frac{e^2\omega_0^2}{6\pi\varepsilon_0 mc^3}}_{\Gamma}\dot{x} + \omega_0^2 x = 0.$$

The solution is the damped wave train

$$x(t) = e^{-\frac{1}{2}\Gamma t}e^{i\omega_0 t}.$$

At angular frequency ω, the Radiation Intensity per angular frequency, $\frac{dI(\omega)}{d\omega}$, is proportional to

$$\left|\int_{t=0}^{t=\infty} x(t)e^{-i\omega t}dt\right|^2,$$

which is proportional to

$$\frac{1}{(\omega-\omega_0)^2+\frac{1}{4}\Gamma^2}.$$

Thus, the spectral line generated by transition of the electron from a higher orbit to a lower one, at the angular frequency $\omega=\omega_0$, is a broadened bell shaped distribution.

The Potential Energy is proportional to

$$(e^{-\frac{1}{2}\Gamma t})^2=e^{-\Gamma t}.$$

The Mean Duration of the radiated pulse is

$$\Delta t=\frac{1}{\Gamma}.$$

And the Mean length of the wave train is

$$\frac{c}{\Gamma}.$$

Applying the Energy-time uncertainty at half the spectral line height,

$$(\underbrace{\Delta E}_{\hbar\Delta\omega})\underbrace{(\Delta t)}_{\frac{1}{\Gamma}}\sim\hbar,$$

$$\Delta\omega=\Gamma.$$

Thus, at half the spectral line height, the frequency width is

$$\Delta\omega=\Gamma,$$

where $\Gamma\dot{x}$ is the radiation reaction term in $\ddot{x}+\Gamma\dot{x}+\omega_0^2x=0$.

References

[Dan1] H. Vic Dannon, The Neutron as a Collapsed-Hydrogen Atom: Zero Point Energy & Nuclear Binding Energy, X Rays & Gamma Rays, Nuclear Forces & Bonding, Neutronic Electrons Orbitals, and Neutron Stars Gauge Institute Journal of Math and Physics, Volume 11, No. 3, August 2015.

[Dan2] H. Vic Dannon, Radiation Equilibrium, Inertia Moments, and the Nucleus Radius in the Electron-Proton Atom Gauge Institute Journal of Math and Physics, Volume 10, No. 3, August 2014.

[Eyges] Leonard Eyges, "*The Classical Electromagnetic Field*", Addison Wesley, 1972.

[Griffiths]David Griffiths, "*Introduction to Electrodynamics*" second edition Prentice hall, 1989.

[Jackson] John Jackson, "*Classical Electrodynamics*", Second Edition, 1975

[Matveyev] Aleksey Matveyev, "*Principles of Electrodynamics*", Reinhold, 1966.

[Milonni] Peter Milonni, "*The Quantum Vacuum: An Introduction to Quantum Electrodynamics*" Academic Press, 1994.

[Ohanian] Hans Ohanian, "*Classical Electrodynamics*" Allyn and Bacon, 1988.

[Oppenheimer], Robert Oppenheimer, "*Lectures on Electrodynamics*". Gordon and Breach, 1970.

[Panofsky] Wolfgang Panofsky; Melba Phillips, "*Classical Electricity and Magnetism*", Second Edition, Addison Wesley, 1962.

[Read] F. H. Read, "*Electromagnetic Radiation*", Wiley, 1980.

[Westgard] James Westgard, "*Electrodynamics: A Concise Introduction*", Springer, 1997.

[Woan] Graham Woan, "*The Cambridge Handbook of Physics Formulas*", Cambridge, 2000.

http://en.wikipedia.org/wiki/Lorentz_transformation

http://en.wikipedia.org/wiki/Relativity_theory

http://en.wikipedia.org/wiki/Hydrogen_atom

http://en.wikipedia.org/wiki/Bohr_model